ORSAY
Série A
N° d'Ordre : 933

PARIJS

ETUDE DES CHAMPS OPTIQUES GAUSSIENS GENERALISES

ARTICLE PRINCIPAL

recouvrant en partie la thèse de Doctorat ès-sciences Physiques

enregistrée au C.N.R.S sous le Numéro A.O. 6745

LE BERRE épouse ROUSSEAU Martine

soutenue le 24 Avril 1973 à l'UNIVERSITE DE PARIS-SUD
Centre d'Orsay

devant le Jury :

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Reprinted from: COHERENCE AND QUANTUM OPTICS
Edited by L. Mandel and E. Wolf
Book available from: Plenum Publishing Corporation
227 West 17th Street, New York, N. Y. 10011

THE APPROACH TO THERMAL EQUILIBRIUM IN SYSTEMS OF COUPLED QUANTUM OSCILLATORS INITIALLY IN A GENERALIZED GAUSSIAN STATE

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The class of generalized Gaussian fields was recently exposed by B. Picinbono and myself[1]. We have shown that the chaotic field is no more than a particular case among other Gaussian fields called generalized Gaussian fields. Among them, the real Gaussian field, or Gaussian field with real amplitude, is especially interesting for many reasons and it has been realized by Perrot and Bendjaballah[5] in our laboratory.

It can be defined by this classical relation:

$$I(t) = x^2(t) \tag{1}$$

where x(t) is a real Gaussian random function. From this relation it follows in particular that the bunching effect is 1.5 times greater for the real Gaussian field than for the chaotic one. The authors cited before[5] have verified many properties related to the real Gaussian field: counting distributions, time intervals, and so on.

Now we shall study some interactions of this real Gaussian field with matter by considering field and matter as a system of quantum oscillators.

The temporal evolution of systems of coupled quantum oscillators has been treated by many authors, especially Louisell, Mollow[4], Robl and Raitford[2].

For solving such a problem we need two sets of parameters:

1. The Hamiltonian

2. The initial conditions.

As it was done by others for reasons of facilities, we limit our problem to quadratic Hamiltonians. That is to say, if a_j^+ and a_k are the creation and annihilation operators for the oscillators j and k, respectively, we shall consider Hamiltonians such as,

$$H = \sum_{i,j} a_i \, a_j \, \alpha_{ij}(t) + \sum_{i,k} a_i^+ \, a_k \, \gamma_{ik}(t) + \sum_i \delta_i(t) a_i + H.C. \quad (2)$$

Such Hamiltonians, in fact, cover almost all the cases encountered, even in nonlinear optics, nowadays.

The P-representation of each quantum oscillator can be determined from the characteristic functions $\chi_j(\xi,\eta,t)$

$$\chi_j(\xi,\eta,t) = Tr_j\{\rho(t)\ e^{-i(\xi Q_j + \eta P_j)}\} = Tr\{\rho(o)\ e^{-i(\xi Q_j(t) + \eta P_j(t))}\} \quad (3)$$

with

$$Q = (a+a^+)\frac{1}{\sqrt{2}}\ , \qquad P = i(a-a^+)\ \frac{1}{\sqrt{2}}\ ,$$

since it is only the Fourier transform of $\chi_j(\frac{\xi}{\sqrt{2}}\ ,\ \frac{\eta}{\sqrt{2}}\ ,\ t)$.

It has been shown by Mollow[4] and Robl[2] that, for such Hamiltonians with $k \neq j$, initially coherent states remain coherent, and initially chaotic oscillators remain in a chaotic state.

The initial state in our case is a real Gaussian one. More precisely, we shall concentrate our attention upon one oscillator in this system, whose initial P-representation is a real Gaussian function of α:

$$P(\alpha,o) = \frac{1}{\sqrt{2\pi}\sigma_x}\ \exp(-\alpha_x^2/2\sigma_x^2)\ \delta(\alpha_y)\ . \quad (4)$$

We suppose that the other oscillators in the system are initially independent and Gaussian (chaotic or real Gaussian):

$$P(\{\alpha_j\},o) = \pi_j P_j(\alpha_j,o)$$

$$P_j(\alpha_j,o) \quad \text{Gaussian functions} . \qquad (4')$$

We can present two conclusions about this problem:

1) *The Gaussian character is maintained.*

2) *The interactions thermalize the system.*

The first one follows immediately from the calculus since the P-representation P_j for each oscillator is only the Fourier transform of

$$\chi_j(\frac{\xi}{\sqrt{2}}, \frac{\eta}{\sqrt{2}}, t) .$$

The second one is not evident at all and perhaps depends upon the model chosen for interactions, but it holds true for the two following examples we have treated.

In particular, we apply the general calculus to the following problems:

The first example concerns the forced oscillation of a mode ω_1 damped by a reservoir and excited by a pump ω. The Schroedinger Hamiltonian of this system is[2]

$$H_S(t) = \hbar\omega_1(a_1^+ a_1, + \frac{1}{2}) + \hbar \sum_\lambda \omega_\lambda(a_{\lambda\mu}^+ a_{\lambda\mu} + \frac{1}{2})$$

$$- \hbar g \sum_{\lambda\mu} (a_1^+ a_{\lambda\mu} + a_1 a_{\lambda\mu}^+) - \hbar G(a_1^+ e^{-i\omega_o t} + a_1 e^{i\omega_o t})$$

where g (resp. G) is the coupling constant between the oscillator ω_1 and the reservoir (resp. the harmonic force).

Let us consider a harmonic oscillator ω_1, which has initially a real Gaussian amplitude. When it is damped by a set of harmonic oscillators, the oscillator ω_1 becomes chaotic whether the damping oscillators are initially real Gaussian or chaotic ones.

The forced oscillation of a Gaussian mode damped by a reservoir is a microscopic explanation of thermalization phenomenon that obeys the law of entropy growth. In particular, this effect may represent the change in the statistical properties of a radiation field after it propagates through different media. Thermalization of solar beams, after they go across the atmosphere, is an analogous phenomenon, although the initial states are not real Gaussian.

The second example concerns the parametric amplification. Generally, it can be described by the following Hamiltonian:

$$H(t) = \frac{\hbar}{2}\omega_a\, a^+(t)\, a(t) + \frac{\hbar}{2}\omega_b\, b^+(t)\, b(t) - \hbar K(a^+(t)b^+(t)e^{-i\omega t}) + \hbar \sum_{\lambda\mu} \omega_\lambda\, a^+_{\lambda\mu}\, a_{\lambda\mu} - \hbar \sum_{\lambda\mu} a^+_{\lambda\mu}\, a_{\lambda\mu} \quad .$$

Two oscillators ω_a and ω_b are coupled by a classical driving force with the sum frequency $\omega = \omega_a + \omega_b$, which exchanges one photon with each oscillator, and two photons with both.

We have shown that when some conditions are fulfilled concerning the mean number of photons in the modes A and B, the mode ω_a becomes chaotic whether it is damped by a reservoir or not.

In particular, if the modes are initially empty, that is to say, if there is no photon, each one becomes chaotic. This is the parametric fluorescence phenomenon.

A particular case of such a parametric amplification with two modes is the stimulated Raman effect: an initially real Gaussian field becomes a scattered chaotic light. In the perspective of an experimental study of the statistical properties of the coherent Raman effect, it should be interesting to introduce a resonant, real Gaussian field in the atomic medium and to study the statistical properties of the stimulated light scattered by the medium.

It is also possible to have parametric amplification when one oscillator ω_a makes two-photon transitions thanks to a pump $\omega=2\omega_a$. This is a degenerate case[2] versus the preceding one because there is only one oscillator. Here the field becomes real Gaussian except for a phase factor; we call it a pseudo-real Gaussian field. This result seems to be due to two-photon interactions between ω_a and the pump. In all other cases each oscillator interacts with others with only one photon.

Thus thermalization appears to be due to one-photon interactions. Recently, Christine Benard[3] showed that, if interactions

in the source are not taken into account, a chaotic source cannot be described as an assembly of atoms emitting independent wave packets. It would be interesting to know if interactions similar to those discussed here, would "thermalize" the electromagnetic field described by Benard.

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